Math 53: Multivariable Calculus **Sections 102, 103**

Worksheet for 2020-04-17

Conceptual Review

Question 1. Qualitatively describe the following parametric surfaces.

(a) $\mathbf{r}(u, v) = \langle u + v, -u, v \rangle$
(b) $\mathbf{r}(u, v) = \langle u^2, -u, v \rangle$
(c) $\mathbf{r}(u, v) = \langle u\cos(2v), v, u\sin(2v) \rangle$ (you may see this surface again on an upcoming written HW)

Question 2. What is one way you could parametrize the graph of a function $f(x, y)$, in other words the surface defined by $z = f(x, y)$?

Question 3. Suppose that all of the u and v grid curves of a parametric surface $\mathbf{r}(u, v)$ are lines. Does it follow that the parametric surface must be a plane?

Problems

Problem 1. Find a normal vector to the plane $\mathbf{r}(u, v) = \langle u + v, u - v, 2u \rangle$ by

(a) finding a Cartesian equation for the plane (i.e. a single equation of x, y, z) and reading off a normal vector from that.
(b) using the $\mathbf{r}_u \times \mathbf{r}_v$ method.

Problem 2. Consider the portion of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 3$. Compute its surface area

(a) using calculus (i.e. techniques from §16.5).
(b) without using calculus. Hint: cut a slit in the cone and unfold it into a “pacman” shape.